

# Collisional Damping of Nuclear Collective Vibrations in a Non-Markovian Transport Approach \*

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## Abstract

A detailed derivation of the collisional widths of collective vibrations is presented in both quantal and semi-classical frameworks by considering the linearized limits of the extended TDHF and the BUU model with a non-Markovian binary collision term. Damping widths of giant dipole and giant quadrupole excitations are calculated by employing an effective Skyrme force, and the results are compared with GDR measurements in Lead and Tin nuclei at finite temperature.

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## I. INTRODUCTION

Excitation energy dependence of the giant resonance is still one of the open problems in the study of collective modes in nuclei at finite temperatures. Recent experimental investigations of the giant dipole resonance in the mass region  $A=110-140$  [1,2] and the  $^{208}\text{Pb}$  nucleus [3] show that damping monotonically increases with excitation energy, and in the former case, saturates at high excitations. In medium-weight and heavy nuclei at relatively low temperatures the overwhelming contribution to damping arises from the spreading width  $\Gamma^\downarrow$  due to mixing of collective states with more complicated states, which is dominated by the coupling with 2p-2h doorway excitations [4–6]. There are essentially two different approaches for calculation of the spreading width  $\Gamma^\downarrow$ : (i) Coherent mechanism due to coupling with low-lying surface modes which provides an important mechanism for damping of giant resonance in particular at low temperatures [7,8], (ii) Damping due to the coupling with incoherent 2p-2h states which is usually referred to as the collisional damping [9,10] and the Landau damping modified by two-body collisions [11,12]. Calculations carried out on the basis of these approaches are partially successful in explaining the broadening of the giant dipole resonance with increasing temperature, but the saturation is still an open problem [13]. In this work, we do not consider the coherent contribution to the spreading width due to the coupling with low-lying surface modes, but investigate in detail the collisional damping at finite temperature due to decay of the collective state into incoherent 2p-2h excitations in the basis of a non-Markovian transport approach.

Semi-classical transport models of the Boltzmann-Uehling-Uhlenbeck (BUU) type are often employed for studying nuclear collective vibrations [14]. Although, these models give a good description for the average resonance energies, they do not provide a realistic description for the collisional relaxation rates. In these standard models, the collision term is treated in a Markovian approximation by assuming that the two-body collisions are local in both space and time, in accordance with Boltzmann's original treatment. This simplification is usually justified by the fact that the duration of two-body collisions is short on the

time scale characteristic of macroscopic evolution of the system. However, when the system possesses fast collective modes with characteristic energies that are not small in comparison with temperature, the standard Markovian treatment is inadequate. It leads to an incorrect energy conservation factor in the collision term, which severely restricts the available 2p-2h phase space for damping of the collective modes. Therefore, for a proper description of the collisional relaxation rates, it is necessary to improve the transport model by including the memory effect due to finite duration of two-body collisions [15–17].

Recently, we have investigated the collisional damping of giant resonances within the linearized limit of the BUU model with a non-Markovian collision term, and derived closed form expressions for damping width of isoscalar and isovector collective vibrations [18,19]. Also, the model has been applied to study the density fluctuations and the growth of instabilities in the nuclear matter within the stochastic Boltzmann-Langevin approach [20,21]. As a result of the non-Markovian structure of two-body collisions, in expressions of transport coefficients of collective modes (i.e. damping width and diffusion coefficient), the available phase space for decay into 2p-2h states is properly taken into account with the correct energy conserving factor. In nuclear matter limit and for isotropic nucleon-nucleon cross-sections, by employing the standard approximation familiar in Fermi Liquid theory, it is possible to give analytical expression for the collisional width as  $\Gamma = \Gamma_0[(\hbar\omega)^2 + (2\pi T)^2]$ , where  $\Gamma_0$  is different for different resonance and determined by nuclear matter properties and in-medium cross-sections. The quadratic temperature dependence fits well with the measured GDR widths in  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  nuclei, however the factor  $\Gamma_0$  calculated with a cross-section of 40 mb underestimates the data by a factor of 2-3. In a recent work, we have performed calculations by employing energy and angle dependent free nucleon-nucleon cross-sections and by taking surface effects into account [22].

In this work, we give a brief description of the non-Markovian extension of the nuclear transport theory in both quantal and semi-classical frameworks, and present a detailed derivation of the collisional widths of collective vibrations. The derivation is carried out in both quantal and semi-classical frameworks by considering linearized limits of the extended

TDHF and the BUU model with memory effects. The major uncertainty in the calculation of collisional widths arises from the lack of an accurate knowledge of the in-medium cross-sections in the vicinity of Fermi energy. The Skyrme force provides a good description of the imaginary part of the single particle optical potential and its radial dependence in the vicinity of Fermi energy [23]. Therefore, it may be suitable for describing in-medium effects in the collision term around Fermi energy. By employing an effective Skyrme force, we perform calculations of the damping widths of the giant dipole and giant quadrupole excitations in semi-classical approximation and compare them with the GDR measurements in Lead and Tin nuclei at finite temperature.

## II. ONE-BODY TRANSPORT MODEL WITH MEMORY EFFECTS

In the extended TDHF approximation, the evolution of the single particle density matrix  $\rho(t)$  is determined by a transport equation [24],

$$i\hbar \frac{\partial}{\partial t} \rho - [h, \rho] = K(\rho) \quad (1)$$

where  $h(\rho)$  is the mean-field Hamiltonian and the quantity on the right-hand-side represents a quantal collision term, which is specified by the correlated part of the two-particle density matrix as  $K(\rho_1) = tr_2[v, C_{12}]$  with  $v$  as the effective residual interactions. The correlated part of the two-particle density matrix is defined as  $C_{12} = \rho_{12} - \widetilde{\rho_1 \rho_2}$ , where  $\widetilde{\rho_1 \rho_2}$  represents the antisymmetrized product of the single-particle density matrices, and it is determined by the second equation of the BBGKY hierarchy. Retaining only the lowest order terms in the residual interactions, the hierarchy can be truncated on the second level, and hence the correlated part of the two-particle density matrix evolves according to,

$$i\hbar \frac{\partial}{\partial t} C_{12} - [h, C_{12}] = F_{12} \quad (2)$$

where

$$F_{12} = (1 - \rho_1)(1 - \rho_2)v\widetilde{\rho_1 \rho_2} - \widetilde{\rho_1 \rho_2}v(1 - \rho_1)(1 - \rho_2). \quad (3)$$

An expression for the collision term can be obtained by formally solving Eq.(2), and substituting the result into Eq.(1),

$$K(\rho) = -\frac{i}{\hbar} \int_0^t d\tau \text{tr}_2[v, G(t, t-\tau) F_{12}(t-\tau) G^\dagger(t, t-\tau)] \quad (4)$$

where

$$G(t, t-\tau) = T \cdot \exp[-\frac{i}{\hbar} \int_{t-\tau}^t dt' h(t')] \quad (5)$$

denotes the mean-field propagator.

The transport equation (1) is usually considered in semi-classical approximation. In this case, one deals with the phase-space density  $f(\mathbf{r}, \mathbf{p})$  defined as the Wigner transform of the density matrix,

$$f(\mathbf{r}, \mathbf{p}) = \int \frac{d\mathbf{k}}{(2\pi\hbar)^3} e^{-i\frac{\mathbf{k}\cdot\mathbf{r}}{\hbar}} \langle \mathbf{p} + \frac{\mathbf{k}}{2} | \rho | \mathbf{p} - \frac{\mathbf{k}}{2} \rangle. \quad (6)$$

Performing the Wigner transform of Eq.(1) and retaining the lowest order terms in gradients in accordance with the standard treatment [25,26], yields a semi-classical transport equation for the phase-space density,

$$\frac{\partial}{\partial t} f(\mathbf{r}, \mathbf{p}) - \{h(f), f(\mathbf{r}, \mathbf{p})\} = K(f). \quad (7)$$

Here, the left hand side describes the Vlasov propagation in terms of the self-consistent one-body Hamiltonian  $h(f)$ , and  $K(f)$  represents the collision term in semi-classical approximation, which has a non-Markovian form due to the memory effects arising from the finite duration of two-body collisions,

$$K(f) = \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \int_0^t d\tau w(12; 34; \tau) [(1-f)(1-f_2)f_3f_4 - ff_2(1-f_3)(1-f_4)]_{t-\tau} \quad (8)$$

In this expression the phase-space density is evaluated at time  $t-\tau$  according to  $f_j(t-\tau) = f(\mathbf{r} - \tau\mathbf{p}_j/m, \mathbf{p}_j + \tau\nabla U, t-\tau)$  and the collision kernel is given by

$$w(12; 34; \tau) = \frac{1}{2\pi} W(12; 34) [g_1(\tau)g_2(\tau)g_3^*(\tau)g_4^*(\tau) + c.c.] \quad (9)$$

where  $g_j(\tau)$  is the Wigner transform of the mean-field propagator

$$g_j(\tau) = \int \frac{d\mathbf{k}}{(2\pi\hbar)^3} e^{-i\frac{\mathbf{k}\cdot\mathbf{r}}{\hbar}} \langle \mathbf{p}_j + \frac{\mathbf{k}}{2} | G(t, t - \tau) | \mathbf{p}_j - \frac{\mathbf{k}}{2} \rangle \quad (10)$$

and  $W(12; 34)$  denotes the basic two-body transition rate

$$W(12; 34) = \frac{\pi}{(2\pi\hbar)^6} | \langle \frac{\mathbf{p}_1 - \mathbf{p}_2}{2} | v | \frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \rangle_A |^2 \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \quad (11)$$

which can be expressed in terms of the scattering cross-section as

$$W(12; 34) = \frac{1}{(2\pi\hbar)^3} \frac{4\hbar}{m^2} \frac{d\sigma}{d\Omega} \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4). \quad (12)$$

The collision term involves different characteristic time scales including the average duration time of two-body collisions  $\tau_d$ , the characteristic time associated with the mean-field fluctuations  $\tau_{mf}$  and the mean-free-time between two-body collisions  $\tau_\lambda$ . The range of the integration over the past history in the collision term (8) (and also in (4)) is essentially determined by the duration time of two-body collisions. Usually, two-body collisions are treated in a Markovian approximation by assuming the duration time of collisions is much shorter than the other time scales  $\tau_d \ll \tau_{mf}, \tau_\lambda$ , which would be appropriate if two-body collisions can be considered instantaneous. In this case, the  $\tau$  dependence of the phase-space density in the collision term can be neglected  $f_j(t - \tau) \approx f_j(t)$ , and the mean-field propagator can be approximated by the kinetic energy alone,  $g_j \approx \exp[-i\tau\epsilon_j]$  with  $\epsilon_j = p_j^2/2m$ . This yields energy conserving two-body collisions, and the resultant semi-classical transport equation is known as the BUU model [27]. The standard description provides a good approximation at intermediate energies when the system does not involve fast collective modes, since the weak-coupling condition is well satisfied due to relatively long mean-free-path of nucleons. When the system possesses fast collective modes, for example high frequency collective vibrations or rapidly growing unstable modes, the Markovian approximation breaks down and the influence of the mean-field fluctuations in the collision term becomes important. The finite duration time allows for a direct coupling between two-body collisions and the mean-field fluctuations, which strongly modifies the collisional relaxation properties of the collective

modes as compared to the Markovian limit, in particular at low temperatures [28]. In this work we consider the mean-field dominated regime in which the nucleon mean-free-time is long as compared to the characteristic time associated with the mean-field fluctuations and the duration time of collisions  $\tau_d, \tau_{mf} \ll \tau_\lambda$ , which maybe referred as weakly non-Markovian regime. In this case, the  $\tau$  dependence of the phase-space density in the collision term can be neglected, as before, and the collision term takes essentially a Markovian form with an effective transition rate given by  $\int_0^t d\tau w(12; 34; \tau)$ . When all different time scales are of the same order of magnitude, the collision term becomes strongly non-Markovian, and the time evolution of the system is accompanied by off-shell two-body collisions .

### III. TRANSPORT DESCRIPTION OF COLLECTIVE VIBRATIONS

We apply the non-Markovian transport model developed in the previous section to describe small amplitude collective vibrations around a stable equilibrium in the linear response approximation, and present an explicit derivation of the expression for the collisional damping widths of the collective modes in both quantal and semi-classical frameworks.

#### A. Quantal treatment

To describe small amplitude collective vibrations around a finite temperature equilibrium state  $\rho_0$ , we linearize Eqs.(1) and (2) for small deviations  $\delta\rho = \rho - \rho_0$  and  $\delta C_{12} = C_{12} - C_{12}^0$ ,

$$i\hbar \frac{\partial}{\partial t} \delta\rho - [\delta h, \rho_0] - [h_0, \delta\rho] = tr_2[v, \delta C_{12}] \quad (13)$$

and

$$i\hbar \frac{\partial}{\partial t} \delta C_{12} - [\delta h, C_{12}^0] - [h_0, \delta C_{12}] = \delta F_{12} \quad (14)$$

where  $\delta h = (\partial U / \partial \rho)_0 \cdot \delta\rho$  represent the small deviations in the single-particle density matrix, the quantity  $\delta F_{12}$  is

$$\begin{aligned}\delta F_{12} = & -\delta\rho_1(1-\rho_2^0)v\widetilde{\rho_1^0\rho_2^0} - (1-\rho_1^0)\delta\rho_2v\widetilde{\rho_1^0\rho_2^0} + \\ & (1-\rho_1^0)(1-\rho_2^0)v\delta\widetilde{\rho_1\rho_2^0} + (1-\rho_1^0)(1-\rho_2^0)v\widetilde{\rho_1^0\delta\rho_2} - h.c.\end{aligned}\quad (15)$$

and the equilibrium correlation function  $C_{12}^0$  is determined by

$$-[h_0, C_{12}^0] = F_{12}^0 \quad (16)$$

with  $F_{12}^0$  as the equilibrium value of  $F_{12}$ .

We can analyze the collective vibration by expanding the small deviations  $\delta\rho$  in terms of normal modes of the system [29],

$$\delta\rho(t) = \sum [z_\lambda(t)\rho_\lambda^\dagger + z_\lambda^*(t)\rho_\lambda] \quad (17)$$

where  $\rho_\lambda^\dagger$  and  $\rho_\lambda$  represent the normal modes of the system. When the damping width is small as compared to the mean frequency of the mode, we can follow a perturbation approach and determine the normal modes by the standard RPA equations without including the collision term,

$$\hbar\omega_\lambda\rho_\lambda^\dagger - [h_\lambda^\dagger, \rho_0] - [h_0, \rho_\lambda^\dagger] = 0. \quad (18)$$

Here,  $\omega_\lambda$  is the frequency of the normal mode and  $h_\lambda^\dagger$  represents the positive frequency part of the vibrating mean-field. It is convenient to introduce the RPA amplitudes  $\hat{O}_\lambda^\dagger$  and  $\hat{O}_\lambda$  associated with normal modes according to  $\rho_\lambda^\dagger = [\hat{O}_\lambda^\dagger, \rho_0]$  and its hermitian conjugate. In the representation which diagonalizes  $\rho_0$ , the RPA amplitudes can be expressed as

$$\langle n|\hat{O}_\lambda^\dagger|m\rangle = \frac{\langle n|h_\lambda^\dagger|m\rangle}{\hbar\omega_\lambda - \epsilon_n + \epsilon_m} \quad (19)$$

and they are normalized as  $tr[\hat{O}_\lambda, \hat{O}_\lambda^\dagger]\rho_0 = 1$ . Substituting the expansion (17) into Eq.(13) and projecting by  $\hat{O}_\lambda$  yields

$$\frac{dz_\lambda}{dt} + i\omega_\lambda z_\lambda = -\frac{1}{2}\Gamma_\lambda z_\lambda \quad (20)$$

for the amplitudes of the normal modes. These amplitudes execute a damped harmonic motion with a damping coefficient given by



$$\Gamma_\lambda = \text{tr}[\hat{O}_\lambda, v]C_\lambda^\dagger \quad (21)$$

and it describes the spreading width of the RPA mode due to coupling with the two particle-two hole states, which is usually referred to as the collisional damping. In this expression,  $C_\lambda^\dagger$  denotes the positive frequency part of the correlations and it is determined by

$$\hbar\omega_\lambda C_\lambda^\dagger - [h_\lambda^\dagger, C_{12}^0] - [h_0, C_\lambda^\dagger] = F_\lambda^\dagger \quad (22)$$

where  $F_\lambda^\dagger$  represents the positive frequency part of  $\delta F_{12}$ . In the representation diagonalizing  $\rho_0$ , the correlation can be expressed as

$$\begin{aligned} \langle nm|C_\lambda^\dagger|kl \rangle &= \frac{\langle nm|[h_\lambda^\dagger, C_{12}^0] + F_\lambda^\dagger|kl \rangle}{\hbar\omega_\lambda - \epsilon_n - \epsilon_m + \epsilon_k + \epsilon_l - i\eta} \\ &= \frac{\langle nm|[\hat{O}_\lambda^\dagger, F_{12}^0] + F_\lambda^\dagger|kl \rangle}{\hbar\omega_\lambda - \epsilon_n - \epsilon_m + \epsilon_k + \epsilon_l - i\eta}. \end{aligned} \quad (23)$$

According to expression (16), the matrix elements of the equilibrium correlation is given by  $\langle nm|C_{12}^0|kl \rangle = \langle nm|F_{12}^0|kl \rangle / (\epsilon_k + \epsilon_l - \epsilon_n - \epsilon_m - i\eta)$ , in which only the principal value part is non-vanishing. The second line of the above expression is obtained by replacing the energy factors in the intermediate states according to  $\delta(\hbar\omega_\lambda - \epsilon_n - \epsilon_m + \epsilon_k + \epsilon_l)$  and using the definition of the RPA amplitudes. Furthermore following the observation

$$[\hat{O}_\lambda^\dagger, F_{12}^0] = -F_\lambda^\dagger + (1 - \rho_1^0)(1 - \rho_2^0)[\hat{O}_\lambda^\dagger, v]\widetilde{\rho_1^0\rho_2^0} - \widetilde{\rho_1^0\rho_2^0}[\hat{O}_\lambda^\dagger, v](1 - \rho_1^0)(1 - \rho_2^0) \quad (24)$$

the correlation can be expressed as,

$$\langle nm|C_\lambda^\dagger|kl \rangle = \langle nm|[\hat{O}_\lambda^\dagger, v]|kl \rangle_A \frac{[\rho_n\rho_m\bar{\rho}_k\bar{\rho}_l - \rho_k\rho_l\bar{\rho}_n\bar{\rho}_m]}{\hbar\omega_\lambda - \epsilon_n - \epsilon_m + \epsilon_k + \epsilon_l - i\eta} \quad (25)$$

where  $\rho_n$  denotes the Fermi-Dirac occupation factor,  $\bar{\rho}_n = 1 - \rho_n$  and  $\langle nm|[\hat{O}_\lambda^\dagger, v]|kl \rangle_A$  represents the anti-symmetric matrix elements. As a result, the damping width of the RPA states is given by [16],

$$\begin{aligned} \Gamma_\lambda &= \frac{\pi}{2} \sum | \langle nm|[\hat{O}_\lambda, v]|kl \rangle_A |^2 \delta(\hbar\omega_\lambda - \epsilon_n - \epsilon_m + \epsilon_k + \epsilon_l) \\ &\quad [\rho_k\rho_l\bar{\rho}_n\bar{\rho}_m - \rho_n\rho_m\bar{\rho}_k\bar{\rho}_l]. \end{aligned} \quad (26)$$

The same expression for the damping width has been derived in ref. [30] by employing a different approach. It also can be obtained in using the Green's function method [31], or more intuitive approaches [7]. It should be mentioned that the expression (26) was written down for the first time by Landau, and it has become a classical result of Fermi Liquid theory. However in the nuclear physics literature several expressions of (26) exist which are at variance with Landau's result [32]. The subtlety hinges on the expression (19) for  $\hat{O}_\lambda^\dagger$ . Indeed, contrary to the ordinary RPA amplitudes, which at zero temperature have only ph or hp components, no phase space factors appear in (19), thus allowing non-zero values of  $\langle n|\hat{O}_\lambda^\dagger|m \rangle$  also for pp and hh configurations (see also expression (30), below).

### B. Semi-classical treatment

It is possible to describe the collective vibrations in semi-classical approximation. In this case, one considers the phase-space density  $\delta f(\mathbf{r}, \mathbf{p})$  associated with small amplitude vibrations. The equation of motion of the small amplitude vibrations in the semi-classical limit is obtained by linearizing the transport equation (7),

$$\frac{\partial}{\partial t}\delta f(\mathbf{r}, \mathbf{p}) + \mathbf{v} \cdot \nabla \delta f(\mathbf{r}, \mathbf{p}) - \mathbf{v} \cdot \nabla \delta h \frac{\partial}{\partial \epsilon} f_0 = \delta K(\mathbf{r}, \mathbf{p}) \quad (27)$$

where  $\delta K(\mathbf{r}, \mathbf{p})$  denotes the linearized collision term,

$$\delta K(\mathbf{r}, \mathbf{p}) = -\frac{i}{\hbar} \int \frac{d\mathbf{k}}{(2\pi\hbar)^3} e^{-i\frac{\mathbf{k}\cdot\mathbf{r}}{\hbar}} d\mathbf{p}_2 \langle \mathbf{p} + \frac{\mathbf{k}}{2}, \mathbf{p}_2 | [v, \delta C_{12}] | \mathbf{p} - \frac{\mathbf{k}}{2}, \mathbf{p}_2 \rangle \quad (28)$$

$\mathbf{v} = \mathbf{p}/m$  and the equilibrium state  $f(\epsilon)$  is taken to be homogeneous for simplicity. In a manner similar to quantal treatment, the phase-space density can be expanded in terms of normal modes as

$$\delta f(\mathbf{r}, \mathbf{p}) = \sum [-i\hbar z_\lambda(t) \mathbf{v} \cdot \nabla O_\lambda^* + i\hbar z_\lambda^*(t) \mathbf{v} \cdot \nabla O_\lambda] \frac{\partial}{\partial \epsilon} f \quad (29)$$

where  $O_\lambda^*$  and  $O_\lambda$  are the Wigner transform of the RPA amplitudes  $\hat{O}_\lambda^\dagger$  and  $\hat{O}_\lambda$ . In the perturbation approach, these amplitudes are given by

$$O_\lambda^*(\mathbf{r}, \mathbf{p}) = \left( \frac{1}{\hbar\omega - i\hbar\mathbf{v} \cdot \nabla} \right) \cdot h_\lambda(\mathbf{r}) \quad (30)$$

and its complex conjugate. In a similar manner, we can expand the correlation function in terms of the normal modes as  $\delta C_{12}(t) = \sum [z_\lambda(t)C_\lambda^\dagger + z_\lambda^*(t)C_\lambda]$ . By inserting this expansion into Eq.(28), we obtain an expression of the collision term in terms of the RPA amplitudes,

$$\delta K(\mathbf{r}, \mathbf{p}) = \frac{1}{2} \sum_\lambda \int d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 W(12; 34) [z_\lambda \Delta O_\lambda^* + z_\lambda^* \Delta O_\lambda] \quad (31)$$

$$[\delta(\hbar\omega_\lambda - \Delta\epsilon) - \delta(\hbar\omega_\lambda + \Delta\epsilon)] [\bar{f}_1 \bar{f}_2 f_3 f_4 - f_1 f_2 \bar{f}_3 \bar{f}_4]$$

where,  $\Delta\epsilon = \epsilon_3 + \epsilon_4 - \epsilon_1 - \epsilon_2$ ,  $\Delta O_\lambda = O_\lambda(3) + O_\lambda(4) - O_\lambda(1) - O_\lambda(2)$  with  $\epsilon(j) = p_j^2/2m$  and  $O_\lambda(j) = O_\lambda(\mathbf{r}, \mathbf{p}_j)$ , and  $W(12; 34)$  is the transition rate given by eqs.(11) or (12). Substituting the normal mode decomposition of the phase-space density into (27) and carrying out a projection with  $O_\lambda$ , we find the expression

$$\Gamma_\lambda = \frac{1}{2} \frac{1}{(2\pi\hbar)^3} \int d\mathbf{r} d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 |\Delta O_\lambda|^2 W(12; 34) \quad (32)$$

$$[\delta(\hbar\omega_\lambda - \Delta\epsilon) - \delta(\hbar\omega_\lambda + \Delta\epsilon)] \bar{f}_1 \bar{f}_2 f_3 f_4$$

for the collisional width, where the normal modes in the semi-classical approximation are normalized according to

$$- \int \frac{i}{(2\pi\hbar)^3} d\mathbf{r} d\mathbf{p} O_\lambda \mathbf{v} \cdot \nabla O_\lambda^* \frac{\partial}{\partial \epsilon} f = 1. \quad (33)$$

We note that, this result for the collisional width can directly be obtained by evaluating the quantal expression (26) in the Thomas-Fermi approximation [33]. We also note that in order to obtain the expressions (26) and (32) for the collisional width in quantal or semi-classical forms, the non-Markovian collision term should be linearized in a consistent manner by including the contributions arising from the mean-field propagator and the phase-space factors. The result is, then, consistent with the Landau's expression for damping of zero sound modes, and also, is in accordance with the quantal fluctuation-dissipation relation [20,21]. If the term involving the mean-field fluctuations (the second term in the left-hand-side of eq.(14)) is ignored, one obtains a wrong expression for the collisional damping which gives a value that is factor of three larger than its correct value in the nuclear matter [18].

It is more convenient to express the semi-classical RPA modes in terms of real functions  $Q_\lambda(\mathbf{r}, \mathbf{p})$  and  $P_\lambda(\mathbf{r}, \mathbf{p})$  defined as

$$Q_\lambda(\mathbf{r}, \mathbf{p}) = \frac{1}{\sqrt{2\omega_\lambda}}[O_\lambda^*(\mathbf{r}, \mathbf{p}) + O_\lambda(\mathbf{r}, \mathbf{p})] \quad (34)$$

and

$$P_\lambda(\mathbf{r}, \mathbf{p}) = i\sqrt{\frac{\omega_\lambda}{2}}[O_\lambda^*(\mathbf{r}, \mathbf{p}) - O_\lambda(\mathbf{r}, \mathbf{p})]. \quad (35)$$

As a result, the normal mode expansion (29) becomes,

$$\delta f(\mathbf{r}, \mathbf{p}) = \sum [q_\lambda(t)\chi_\lambda^q(\mathbf{r}, \mathbf{p}) + p_\lambda(t)\chi_\lambda^p(\mathbf{r}, \mathbf{p})](-\frac{\partial}{\partial \epsilon}f) \quad (36)$$

where  $\chi_\lambda^q = -\hbar \mathbf{v} \cdot \nabla P_\lambda$  and  $\chi_\lambda^p = -\hbar \mathbf{v} \cdot \nabla Q_\lambda$  represent the distortion factors of the phase-space density associated with the real variables  $q_\lambda = \frac{1}{\sqrt{2\omega_\lambda}}[z_\lambda^* + z_\lambda]$  and  $p_\lambda = i\sqrt{\frac{\omega_\lambda}{2}}[z_\lambda^* - z_\lambda]$ , respectively. In the collision term (31), the factor  $z_\lambda \Delta O_\lambda^* + z_\lambda^* \Delta O_\lambda$  is replaced by

$$\begin{aligned} z_\lambda \Delta O_\lambda^* + z_\lambda^* \Delta O_\lambda &= i\frac{z_\lambda}{\omega_\lambda}(\mathbf{v} \cdot \nabla) \Delta O_\lambda^* - i\frac{z_\lambda^*}{\omega_\lambda}(\mathbf{v} \cdot \nabla) \Delta O_\lambda \\ &= \frac{1}{\hbar \omega_\lambda} [q_\lambda \chi_\lambda^q(\mathbf{r}, \mathbf{p}) + p_\lambda \chi_\lambda^p(\mathbf{r}, \mathbf{p})] \end{aligned} \quad (37)$$

where the first line follows from an identity satisfied by the semi-classical RPA amplitudes,  $O_\lambda^*(\mathbf{r}, \mathbf{p}) = [h_\lambda(\mathbf{r}) + i\hbar \mathbf{v} \cdot \nabla O_\lambda^*(\mathbf{r})]/\hbar \omega_\lambda$ . In order to deduce the equations for the real variables  $q_\lambda(t)$  and  $p_\lambda(t)$ , we substitute the expansion (36) into Eq.(27) and project the resultant equation by  $O_\lambda$  and  $P_\lambda$ , or equivalently by  $\chi_\lambda^q$  and  $\chi_\lambda^p$ . This gives two coupled equations for  $q_\lambda(t)$  and  $p_\lambda(t)$ , which can be combined to yield an equation in the form of a damped harmonic oscillator,

$$\ddot{q}_\lambda + \left( \omega_\lambda^2 + \left( \frac{\Gamma_\lambda}{2\hbar} \right)^2 \right) q_\lambda = -\frac{\Gamma_\lambda}{\hbar} \dot{q}_\lambda \quad (38)$$

where the collisional width is given by

$$\begin{aligned} \Gamma_\lambda &= \frac{1}{(2\pi\hbar)^3} \int d\mathbf{r} d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 [(\Delta \chi_\lambda^q)^2 + (\Delta \chi_\lambda^p)^2] W(12; 34) \\ &\quad \left[ \frac{\delta(\hbar\omega_\lambda - \Delta\epsilon) - \delta(\hbar\omega_\lambda + \Delta\epsilon)}{4\hbar\omega_\lambda} \right] \bar{f}_1 \bar{f}_2 f_3 f_4 \end{aligned} \quad (39)$$

with the distortion factors normalized according to

$$\int \frac{1}{(2\pi\hbar)^3} d\mathbf{r} d\mathbf{p} (\chi_\lambda^q)^2 \left(-\frac{\partial}{\partial \epsilon} f\right) = \int \frac{1}{(2\pi\hbar)^3} d\mathbf{r} d\mathbf{p} (\chi_\lambda^p)^2 \left(-\frac{\partial}{\partial \epsilon} f\right) = 1. \quad (40)$$

This expression, which is equivalent to the one given by Eq.(32), provides a useful formula to calculate collisional damping in terms of distortion factors of the momentum distribution associated with the collective modes. The distortion factors may be determined from the RPA treatment, or can be directly parametrized on physical grounds. In practice, only one of the factors,  $\chi_\lambda^q$  or  $\chi_\lambda^p$  which is associated with a distortion of the momentum distribution, contributes the collisional damping.

Spin-isospin effects in collective vibration can be easily incorporated in the semi-classical RPA treatment by considering proton and neutron degrees of freedom separately. The small deviations of the phase-space densities  $\delta f_p(\mathbf{r}, \mathbf{p})$ ,  $\delta f_n(\mathbf{r}, \mathbf{p})$  of protons and neutrons are determined by two coupled equations analogous to Eq.(27). The collision terms in these equations involve binary collisions between proton-proton, neutron-neutron and proton-neutron, and a summation over the spins of the colliding particles. Observing that in isoscalar/isovector modes protons and neutrons vibrate in-phase/out-of phase,  $\delta f_p(\mathbf{r}, \mathbf{p}) = \mp \delta f_n(\mathbf{r}, \mathbf{p})$ , we can deduce equation of motions for describing isoscalar/isovector vibrations by adding and subtracting the corresponding equations for protons and neutrons. Carrying out the semi-classical RPA treatment presented above, we obtain  $\Gamma_\lambda = \int d\mathbf{r} \Gamma_\lambda(r)$  with

$$\Gamma_\lambda^s(r) = \frac{1}{N_\lambda} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 [W_{pp} + W_{nn} + 2W_{pn}] \left(\frac{\Delta\chi_\lambda}{2}\right)^2 Z f_1 f_2 \bar{f}_3 \bar{f}_4 \quad (41)$$

and

$$\Gamma_\lambda^v = \frac{1}{N_\lambda} \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 [(W_{pp} + W_{nn}) \left(\frac{\Delta\chi_\lambda}{2}\right)^2 + 2W_{pn} \left(\frac{\widetilde{\Delta\chi}_\lambda}{2}\right)^2] Z f_1 f_2 \bar{f}_3 \bar{f}_4 \quad (42)$$

for the collisional widths of isoscalar and isovector modes, respectively. Here,  $N_\lambda = \int d\mathbf{r} d\mathbf{p} (\chi_\lambda)^2 (-\frac{\partial}{\partial \epsilon} f)$  is a normalization,  $\Delta\chi_\lambda = \chi_\lambda(1) + \chi_\lambda(2) - \chi_\lambda(3) - \chi_\lambda(4)$ ,  $\widetilde{\Delta\chi}_\lambda = \chi_\lambda(1) - \chi_\lambda(2) - \chi_\lambda(3) + \chi_\lambda(4)$ , and  $Z = [\delta(\hbar\omega_\lambda - \Delta\epsilon) - \delta(\hbar\omega_\lambda + \Delta\epsilon)]/\hbar\omega_\lambda$ . In these expressions, transition rates associated with proton-proton, neutron-neutron and proton-neutron collisions are given by eq.(12) with the corresponding cross-sections

$$\left(\frac{d\sigma}{d\Omega}\right)_{pp} = \left(\frac{d\sigma}{d\Omega}\right)_{nn} = \frac{\pi}{(2\pi\hbar)^3} \frac{m^2}{4\hbar} \cdot \frac{1}{4} \sum_S (2S+1) |< \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}; S, T = 1 | v | \frac{\mathbf{p}_3 - \mathbf{p}_4}{2}; S, T = 1 >_A|^2 \quad (43)$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{pn} = \frac{\pi}{(2\pi\hbar)^3} \frac{m^2}{4\hbar} \cdot \frac{1}{8} \sum_{S,T} (2S+1) |< \frac{\mathbf{p}_1 - \mathbf{p}_2}{2}; S, T | v | \frac{\mathbf{p}_3 - \mathbf{p}_4}{2}; S, T >_A|^2 \quad (44)$$

where  $< \mathbf{p}_1 - \mathbf{p}_2/2; S, T | v | \mathbf{p}_3 - \mathbf{p}_4/2; S, T >_A$  represents the fully antisymmetric two body matrix element of the residual interaction between states with total spin and isospin  $S$  and  $T$ . The residual interactions  $v$  should be understood as an effective density dependent force. It can indeed be shown that a reasonable approximation for  $v$  is the so-called  $G$ -matrix [29]. Microscopic  $G$ -matrices are not very practical for explicit use, and thus we adopt below a more phenomenological point of view replacing the  $G$ -matrix by one of the more recent Skyrme forces. In doing so, we, however, should be carefull, since in the vicinity of nuclear surface Skyrme-type forces usually do not match at all to the free nucleon-nucleon cross-sections.

#### IV. DAMPING OF GD AND GQ EXCITATIONS

We apply the formulas (41) and (42) to calculate the collisional widths of the giant quadruple and dipole modes by parametrizing the distortion factors of the momentum distribution in terms of Legendre functions as  $\chi_Q = p^2 P_2(\cos\theta)$  and  $\chi_D = p P_1(\cos\theta)$ . In our calculations, we employ an effective Skyrme force, which is parameterized as

$$v = t_0(1 + x_0 P_\sigma) \delta(\mathbf{r}_1 - \mathbf{r}_2) + \frac{t_1}{2} [\delta(\mathbf{r}_1 - \mathbf{r}_2) \hat{\mathbf{k}}^2 + \hat{\mathbf{k}}'^2 \delta(\mathbf{r}_1 - \mathbf{r}_2)] + t_2 \hat{\mathbf{k}}' \cdot \delta(\mathbf{r}_1 - \mathbf{r}_2) \hat{\mathbf{k}} + \frac{t_3}{6} \rho^\alpha \delta(\mathbf{r}_1 - \mathbf{r}_2) \quad (45)$$

where  $\hat{\mathbf{k}} = (\mathbf{p}_1 - \mathbf{p}_2)/2\hbar$  represents the relative momentum operator with  $\hat{\mathbf{k}}$  is acting to right and  $\hat{\mathbf{k}}'$  is acting to left. In the case of the quadruple mode the collisional width is determined by the spin-isospin averaged nucleon-nucleon cross-section,  $(d\sigma/d\Omega)_0 =$

$[(d\sigma/d\Omega)_{pp} + (d\sigma/d\Omega)_{nn} + 2(d\sigma/d\Omega)_{pn}]/4$ . In the case of the dipole mode the only contribution comes from the spin averaged proton-neutron cross-section,  $(d\sigma/d\Omega)_{pn}$ . In terms of the effective Skyrme force these cross-sections are given by

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\pi}{(2\pi\hbar)^3} \frac{m^{*2}}{4\hbar} \left( \frac{3}{4}[t_0(1-x_0) + \frac{t_1}{2}(\mathbf{k}^2 + \mathbf{k}'^2) + \frac{t_3}{6}\rho^\alpha]^2 + \frac{5}{2}[t_2\mathbf{k} \cdot \mathbf{k}']^2 + \frac{3}{4}[t_0(1+x_0) + \frac{t_1}{2}(\mathbf{k}^2 + \mathbf{k}'^2) + \frac{t_3}{6}\rho^\alpha]^2 \right) \quad (46)$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{pn} = \frac{\pi}{(2\pi\hbar)^3} \frac{m^{*2}}{4\hbar} \left( \frac{1}{2}[t_0(1-x_0) + \frac{t_1}{2}(\mathbf{k}^2 + \mathbf{k}'^2) + \frac{t_3}{6}\rho^\alpha]^2 + 2[t_2\mathbf{k} \cdot \mathbf{k}']^2 + \frac{3}{2}[t_0(1+x_0) + \frac{t_1}{2}(\mathbf{k}^2 + \mathbf{k}'^2) + \frac{t_3}{6}\rho^\alpha]^2 \right) \quad (47)$$

where  $\mathbf{k} = (\mathbf{p}_1 - \mathbf{p}_2)/2\hbar$  and  $\mathbf{k}' = (\mathbf{p}_3 - \mathbf{p}_4)/2\hbar$  are the relative momenta before and after the binary collision, and  $m^*$  denotes the effective mass

$$\frac{1}{m^*(r)} = \frac{1}{m} \left[ 1 + \frac{2m}{\hbar^2} \frac{1}{16} (3t_1 + 5t_2) \rho(r) \right]. \quad (48)$$

In the bulk of the nucleus the Pauli blocking is very effective, and hence, the overwhelming contributions to momentum integrals in expressions (41) and (42) arise in the vicinity of the Fermi surface. We can approximately perform these integrals by employing the standard coordinate transformation [34],

$$\int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_4 \delta(\mathbf{p}_1 + \mathbf{p}_2 - \mathbf{p}_3 - \mathbf{p}_4) \cdots \approx \int p_F \frac{m^{*4}}{2} d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \frac{d\Omega_1 d\Omega}{\cos\theta/2} d\phi_2 \cdots \quad (49)$$

Furthermore, for temperatures small compared to the Fermi energy,  $T \ll \epsilon_F$ , the energy integrals can be calculated analytically using the formula [34,35],

$$\int d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4 \delta(\hbar\omega \pm \Delta\epsilon) \bar{f}_1 \bar{f}_2 f_3 f_4 \approx \mp \frac{\hbar\omega}{6} \frac{(\hbar\omega)^2 + (2\pi T)^2}{1 - \exp(-\hbar\omega/T)}. \quad (50)$$

Then, the bulk contribution to the collisional widths of the quadrupole and dipole modes can be expressed as,

$$\Gamma_Q^{bulk}(r) = \frac{\hbar}{N_Q} \frac{4\pi}{5} m^{*2} \rho p_F^2 I_Q(r) [(\hbar\omega)^2 + (2\pi T)^2] \quad (51)$$

and

$$\Gamma_D^{bulk}(r) = \frac{\hbar}{N_D} \frac{2\pi}{3} m^{*2} \rho I_D(r) [(\hbar\omega)^2 + (2\pi T)^2]. \quad (52)$$

Here,  $\rho$  denotes the particle density,  $\rho = (4/(2\pi\hbar)^3)(4\pi/3)p_F^3$ , the normalizations are  $N_Q = (4\pi/5) \int d\mathbf{r} m^* p_F^5$ ,  $N_D = (4\pi/3) \int d\mathbf{r} m^* p_F^3$ , and the quantities  $I_Q$ ,  $I_D$  are given by

$$I_Q = \int \sin \frac{\theta}{2} d\theta d\phi [1 + P_2(\cos \theta) - 2P_2(\cos \theta'_3)] \left( \frac{d\sigma}{d\Omega} \right)_0 \quad (53)$$

and

$$I_D = \int \sin \frac{\theta}{2} d\theta d\phi [1 - P_1(\cos \theta)] \left( \frac{d\sigma}{d\Omega} \right)_{pn}. \quad (54)$$

In these expressions, the angular integrals can be performed analytically by noting that in the vicinity of Fermi surface the momentum dependent terms in the cross-sections can be expressed in terms of the standard variables as  $\mathbf{k} \cdot \mathbf{k}' = -k_F^2 \sin^2 \theta / 2 \cos \phi$  and  $\mathbf{k}^2 = \mathbf{k}'^2 = k_F^2 \sin^2 \theta / 2$ , and  $\cos \theta'_3 = (\cos \theta / 2)^2 - (\sin \theta / 2)^2 \cos \phi$ . As already mentioned earlier, the cross-sections based on Skyrme forces have a strongly erroneous behaviour at very low densities. For this reason, we can not use expressions (51) and (52) far out in the surface. It is therefore absolutely necessary to develop effective forces which have the correct free cross-section limit. For the time being, we develop an interpolation scheme. In the vicinity of nuclear surface,  $\rho(r) \ll \rho_0$ , the Pauli blocking is not effective. In this case, it is convenient to transform the integration variables in (41) and (42) into the total momenta  $\mathbf{P} = \mathbf{p}_1 + \mathbf{p}_2$ ,  $\mathbf{P}' = \mathbf{p}_3 + \mathbf{p}_4$  and relative momenta  $\mathbf{q} = (\mathbf{p}_1 - \mathbf{p}_2)/2$ ,  $\mathbf{q}' = (\mathbf{p}_3 - \mathbf{p}_4)/2$  before and after the collision. Due to the energy conservation, the magnitude of the relative momentum after the collision is restricted according to  $q' = \sqrt{q^2 \mp m^* \hbar \omega}$ . In the tail region for  $\epsilon_F, T \ll \hbar \omega$ , the expressions (41) and (42) may be estimated by omitting the Pauli blocking factors and neglecting the  $q$ -dependent terms. This gives,

$$\Gamma_Q^{surf}(r) \approx \frac{\hbar}{N_Q} \frac{2\pi}{3} \rho p_F^3 (m^* \hbar \omega)^{3/2} I_Q(r) \quad (55)$$

and



$$\Gamma_D^{surf}(r) \approx \frac{\hbar}{N_D} \frac{\pi}{3} \rho p_F^3 (m^* \hbar \omega)^{1/2} I_D(r). \quad (56)$$

We define an effective Pauli blocking factor as the ratio of the damping width with and without the Pauli blocking factors in expressions (41) and (42),  $F_\lambda = \Gamma_\lambda(r)/\Gamma_\lambda^{noPauli}(r)$ , for  $\lambda = Q$  or  $D$ , and parameterize it in the following form,

$$F_\lambda(r) = 1 + \left( \frac{\epsilon_F(r)}{\epsilon_F(0)} \right)^\beta (F_\lambda(0) - 1) \quad (57)$$

where  $\epsilon_F(0)$  is the Fermi energy at the bulk corresponding to the central density  $\rho_0$ ,  $F_\lambda(0)$  is the effective factor at the bulk and  $\beta = \hbar\omega_\lambda/2\epsilon_F(0)$ . As a function of  $r$ , the effective blocking factor remains essentially constant and equals to its bulk value until the density reaches about 1/2 of the central density, and then it smoothly goes to one at the surface of the nucleus. This form provides a good approximation for the exact calculations of the effective Pauli blocking of  $2p - 1h$  excitations in connection with the collisional damping of single particle states as reported in [36]. Therefore we expect, it provides a reasonable approximation for  $2p - 2h$  excitations, and calculate the collisional widths in an approximate manner by smoothly joining the bulk contribution with the surface contribution in accordance with the approximate blocking factor (57),

$$\Gamma_Q = \int d\mathbf{r} \left( \Gamma_Q^{bulk}(r) \left( \frac{\epsilon_F(r)}{\epsilon_F(0)} \right)^{\beta+0.5} + \Gamma_Q^{surf}(r) \left[ 1 - \left( \frac{\epsilon_F(r)}{\epsilon_F(0)} \right)^\beta \right] \right) \equiv \int d\mathbf{r} \Gamma_Q(r) \quad (58)$$

and

$$\Gamma_D = \int d\mathbf{r} \left( \Gamma_D^{bulk}(r) \left( \frac{\epsilon_F(r)}{\epsilon_F(0)} \right)^{\beta+1.5} + \Gamma_D^{surf}(r) \left[ 1 - \left( \frac{\epsilon_F(r)}{\epsilon_F(0)} \right)^\beta \right] \right) \equiv \int d\mathbf{r} \Gamma_D(r) \quad (59)$$

We determine the nuclear density in the Thomas-Fermi approximation using a Woods-Saxon potential with a depth  $V_0 = -44$  MeV, thickness  $a = 0.67$  fm and sharp radius  $R_0 = 1.27A^{1/3}$  fm. We perform the calculations with the SkM force with parameters  $\alpha = 1/6$ ,  $x_0 = 0.09$ ,  $t_0 = -2645 \text{ MeV fm}^3$ ,  $t_1 = 410 \text{ MeV fm}^5$ ,  $t_2 = -135 \text{ MeV fm}^5$ , and  $t_3 = 15,595 \text{ MeV fm}^{7/2}$ . For the mass dependence of the resonance energies for spherical medium mass and heavy nuclei we use the formulas,  $\hbar\omega = 64A^{-1/3}$  MeV for GQR and  $\hbar\omega = 80A^{-1/3}$

MeV for GDR. Figures 1 and 2 illustrate the relative contribution of the damping widths of GDR and GQR for a nucleus with  $A = 120$  as function of  $r$ . In these figures and also in the other figures, the dashed and dotted lines show the result of the calculations with the effective mass  $m^*$  and the bare mass  $m$ , respectively. The sharp rise of  $\Gamma(r)$  in the vicinity of the surface is largely due to the effective mass, which is small in the bulk and approaches to its bare value at the surface, and to a lesser extend due to the increase of the Skyrme cross-section at low densities. For comparison, the results for constant cross-sections of  $\sigma_0 = 30$  mb and  $\sigma_{pn} = 40$  mb are shown in the same figures by solid lines. These constant cross-sections correspond to a zero range force with a strength  $t_0 = -300 \text{ MeV fm}^3$  and all other parameters are set equal to zero in (45). Figures 3 and 4 show the atomic mass dependence of the GDR and GQR widths and comparison with data, respectively. The SkM force with the effective mass underestimates the average trend of GDR for medium weight and heavy nuclei by about a factor two. The calculations with the bare mass gives a better description of the average trend. In the GQR-case, the discrepancy between the calculations and the average trend of data is larger than in the GDR-case. In figures 5 and 6, the measured GDR widths in  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  nuclei are plotted as a function of temperature, and compared with the calculations performed with the effective mass and the bare mass shown by dashed and dotted lines, respectively. The calculations with the effective mass provide a reasonable description of the temperature dependence of data, but the magnitude of damping is underestimated in both cases. The calculation with the bare mass give larger damping, but the damping widths appear to grow faster than data as a function of temperature.

## V. CONCLUSIONS

In the standard nuclear transport models (mean-field transport models and their stochastic extensions in semi-classical or quantal form), the binary collisions are treated in a Markovian approximation by assuming the duration time of collision is much shorter than the

mean-field fluctuations and the mean-free-time between collisions, which would be appropriate if two-body collisions can be considered instantaneous. As a result, the standard model provides a classical description of transport properties of collective motion that is valid at low frequency-high temperature limit. When the system possesses fast collective modes, the standard description breaks down and it is necessary to incorporate memory effect associated with the finite duration of binary collisions. This yields a non-Markovian extension of the transport description in which the basic transition rate is modified by involving a direct coupling between collective modes and two-body collisions. The extended model leads to a description of the transport properties of collective modes that is in accordance with the quantal fluctuation-dissipation relation. In this work, we present a detailed derivation of the collisional widths of isoscalar and isovector collective nuclear vibrations in both quantal and semi-classical frameworks by considering the linearized limits of the extended TDHF and the BUU model with non-Markovian collision term. The standard treatment with a Markovian collision term leads to vanishing collisional widths at zero temperature. Whereas in the non-Markovian treatment the collisional widths are finite and consistent with the Landau's expression for damping of zero sound in Fermi liquids. The numerical result of the collisional damping is rather sensitive to the in-medium nucleon-nucleon cross-sections around Fermi energy, for which an accurate information is not available. In the present investigations, by employing an effective Skyrme force with SkM parameters, we carry out calculations of the damping widths of giant quadrupole and giant dipole excitations in semi-classical framework, and compare the results with the GDR measurements in  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  nuclei at finite temperatures. In particular for GDR, the magnitude of the collisional damping with the bare nucleon mass is a sizable fraction of the observed damping widths at zero temperature, however the effective mass further reduces the magnitude of damping in both cases. Aside from the magnitude, calculations are qualitatively in agreement with the broadening of GDR widths as a function of temperature in both  $^{120}\text{Sn}$  and  $^{208}\text{Pb}$  nuclei.

One of the main aims of the present investigation was to assess how much of the the total width of GRE's is exhausted by decay into the incoherent 2p-2h states. The calculations have

been performed within the Thomas-Fermi approximation, which is known from independent studies to be very reliable for description of the 2p-2h level densities [23]. However, our results remain semi-quantitative, since in the Thomas-Fermi framework we need the in-medium cross-sections locally down to very low densities, i.e. we need cross-sections which interpolate correctly between the free space and the medium. At the moment such cross-sections are not available (at least not analytically), and thus we were forced to invent our own interpolation scheme, which, although reasonable, is subject to some uncertainties. We found that a sizeable fraction of  $\Gamma^\downarrow$  is accounted for by the incoherent decay. This is the case, for instance, for the GDR and also to a lesser extend for the GQR. In addition, we found for the GDR that the percentage of the incoherent decay, depending somewhat on the nucleon effective mass, strongly increases with temperature. This finding is not very surprising, since at temperature  $T > 3$  MeV shell effects are absent and the collectivity of the vibrational states is strongly reduced. Therefore to a good approximation, a hot nucleus can be regarded as a finite blob of a hot Fermi gas. In spite of this, in particular at lower temperatures, the influence of the low-lying collective states on the damping is missing in our description. Also, the question of the saturation of the GDR width has not been addressed in this work. Further studies are needed for a quantitative description of the damping of nuclear Giant Resonances.

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### Figure Captions:

**Figure 1:** The relative contribution to the collisional damping width of GDR as a function of  $r$  for  $A = 120$  at zero temperature. Solid, dashed and dotted lines are calculations with a constant cross-section  $\sigma_{pn} = 40$  mb, with the SkM force with the effective mass and with the SkM force with the bare mass, respectively.

**Figure 2:** The relative contribution to the collisional damping width of GQR as a function of  $r$  for  $A = 120$  at zero temperature. Solid, dashed and dotted lines are calculations with a constant cross-section  $\sigma_{pn} = 30$  mb, with the SkM force with the effective mass and with the SkM force with the bare mass, respectively.

**Figure 3:** The collisional damping width of GDR as a function of mass number  $A$  at zero temperature. Solid, dashed, dotted lines are calculations with a constant cross-section  $\sigma_{pn} = 40$  mb, with the SkM force with the effective mass, with the SkM force with the bare mass, respectively, and the points show the data.

**Figure 4:** The collisional damping width of GQR as a function of mass number  $A$  at zero temperature. Solid, dashed, dotted lines are calculations with a constant cross-section  $\sigma_{pn} = 30$  mb, with the SkM force with the effective mass, with the SkM force with the bare mass, respectively, and the points show the data.

**Figure 5:** The collisional damping width of GDR in  $^{120}\text{Sn}$  as a function of temperature. Dashed, dotted lines and points are calculations with the SkM force with the effective mass, with the SkM force with the bare mass and data taken from [2], respectively.

**Figure 6:** The collisional damping width of GDR in  $^{208}\text{Pb}$  as a function of temperature. Dashed, dotted lines and points are calculations with the SkM force with the effective mass, with the SkM force with the bare mass and data taken from [3], respectively.

Figure 1

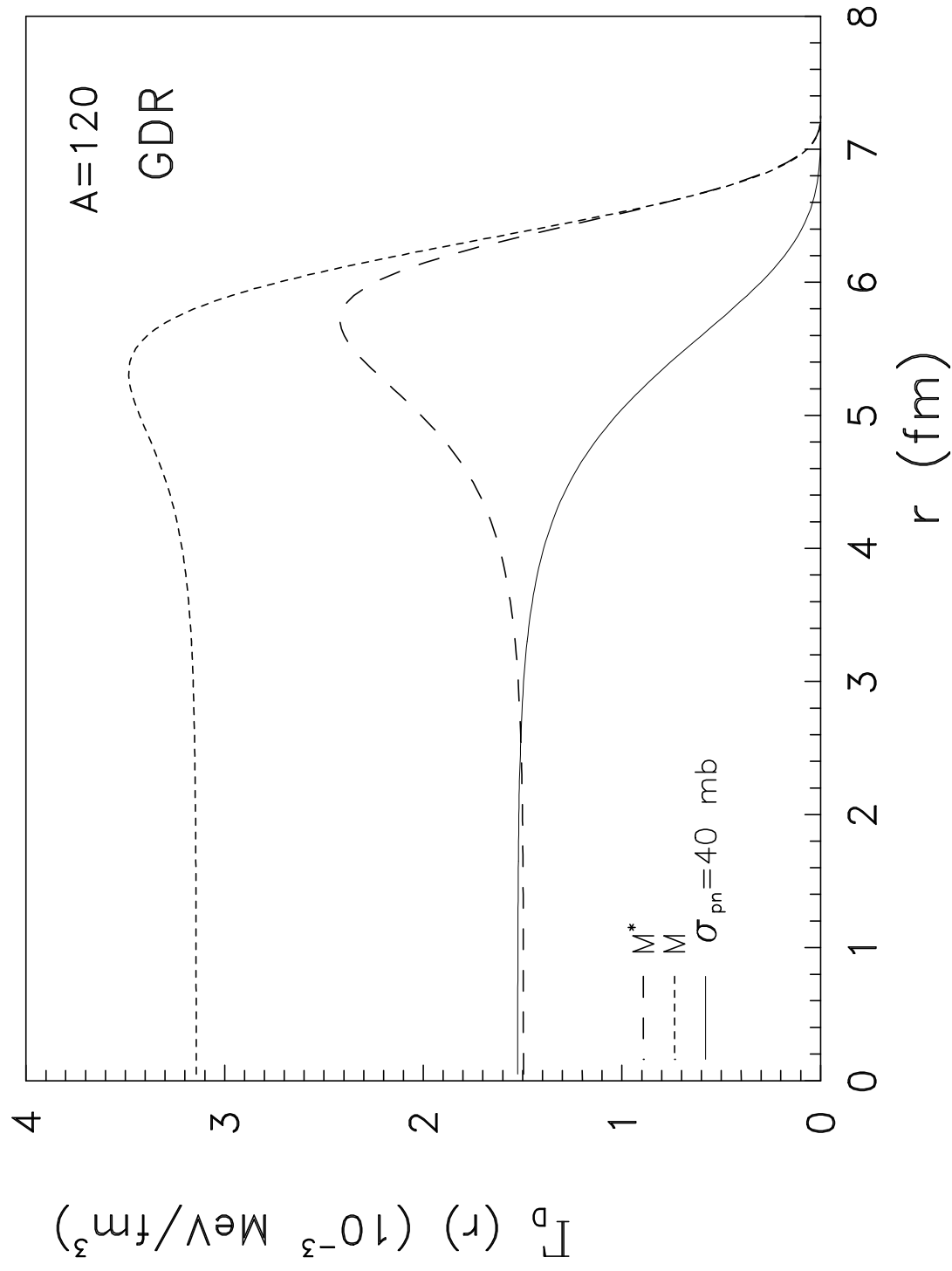




Figure 2

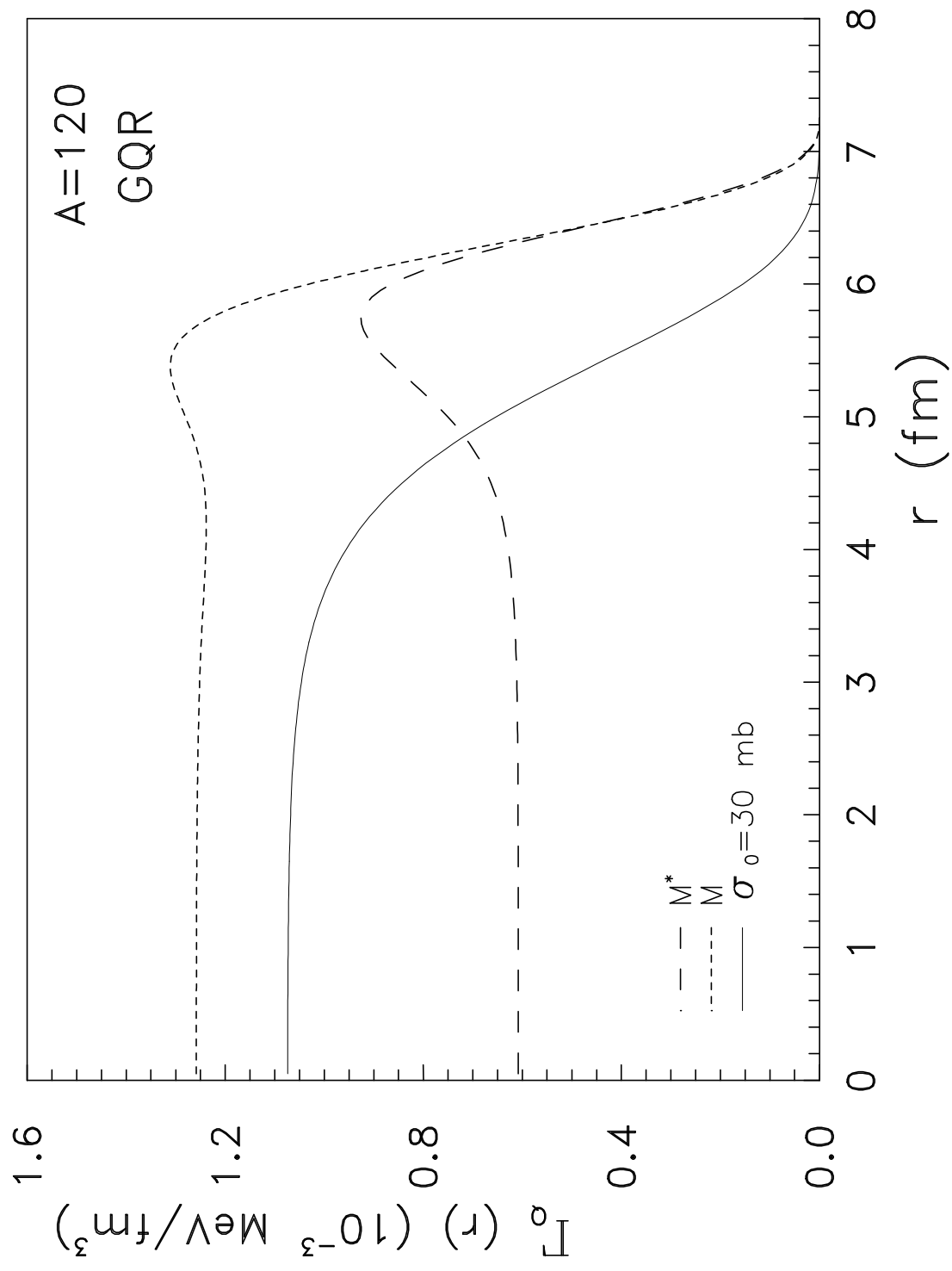


Figure 3

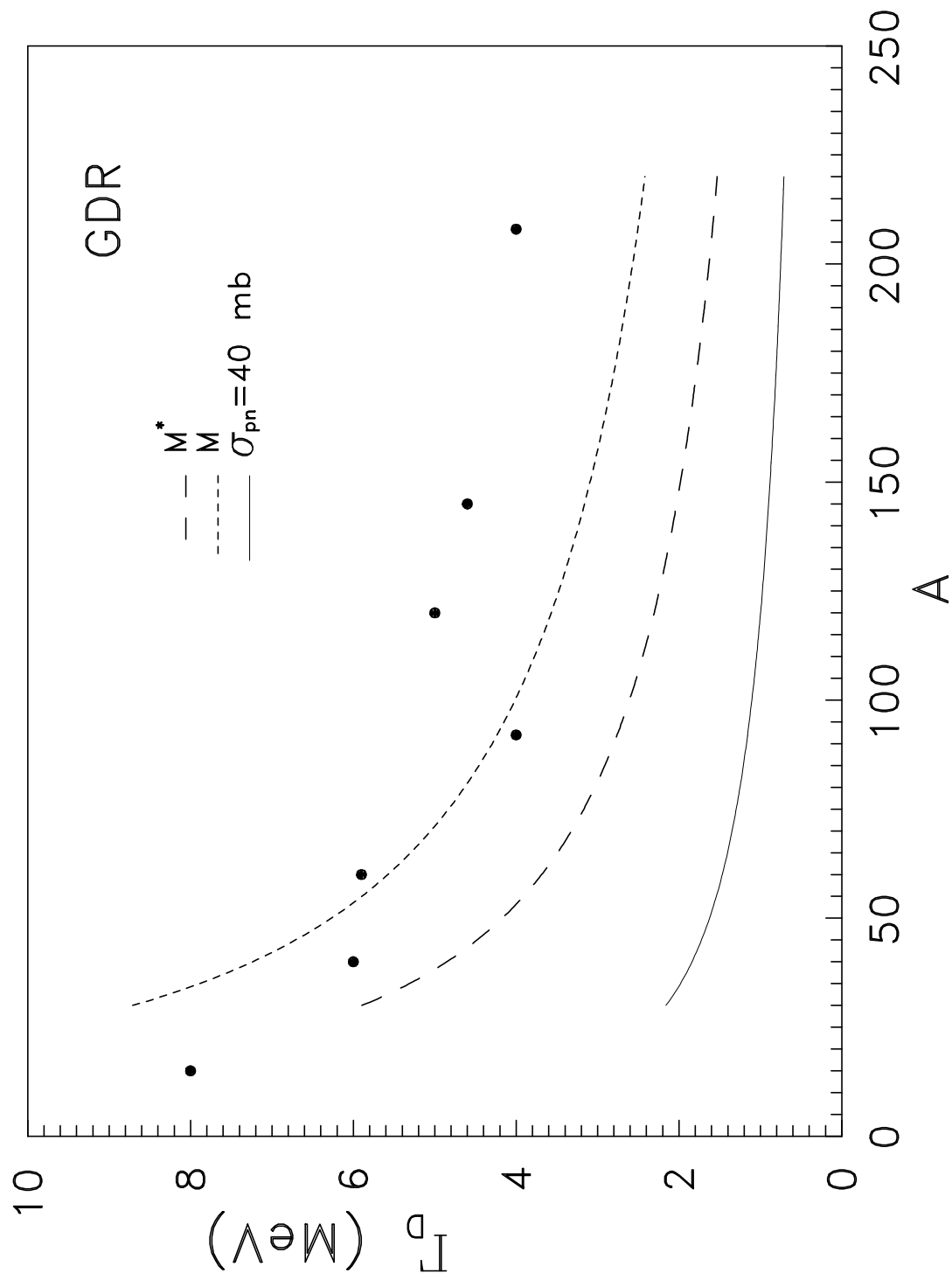


Figure 4

